Comparison of Jevons and Carli elementary price indices

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Abstract

Most of countries use either Jevons or Carli index for the calculation of their Consumer Price Index (CPI) at the lowest (elementary) level of aggregation. The choice of the elementary formula for the inflation measurement does matter and the effect of the change of the index formula was estimated by the Bureau Labor Statistics (2001). It was shown (Hardy et. al, 1934) that the difference between the Carli index and the Jevons index is bounded from below by the variance of the price relatives. In this paper we extend this result comparing expected values of these sample indices under the assumption that prices are described by geometric Brownian motion.

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JEL Classification: E1, E2, E3

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1 Introduction

Elementary price indices are used in inflation measurement on the lowest level of aggregation. Choice of the elementary formula does matter. For instance, in January 1999 the formula used for aggregating price changes for the US consumer price index (CPI) at the lower level of aggregation was changed into a ratio of geometric means of prices (Silver and Heravi, 2007). The effect of this change was researched by the Bureau of Labor Statistics (2001) to reduce the annual rate of increase in the CPI by approximately 0.2 percentage points. As a consequence, it increased a cumulative national debt from over-indexing the federal budget by more than $200 billion per twelve years (Boskin et al., 1996, 1998).

In March 2013, the UK's Office for National Statistics (ONS) started to publish a new inflation index – RPIJ. This index is identical to the Retail Price Index (RPI), except it uses a geometric mean of price relatives (known as Jevons index) rather than an arithmetic mean of price relatives (the Carli index). Moreover, none of the 28 European Union countries makes use of the Carli index in their national price indices. Eurostat regulations do not allow the use of the Carli index in the construction of members' Harmonized Index of Consumer Prices (HICP). There has been a general trend in replacing the Carli index with the Jevons or the Dutot formulas (Evans, 2012). Some countries abandoned the Carli index formula in favour of other price indices over the last few decades, like Canada (in 1978), Luxemburg (in 1996),

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Australia (in 1998), Italy (in 1999) or Switzerland (in 2000). In 1996, the Boskin Commision in the USA recommended that a Carli-like index that was used in the US CPI should be replaced with the Jevons index (Levell, 2015).

There are many papers that compare the above-mentioned unweighted price index numbers (Silver and Heravi, 2007; Levell, 2015). In this paper we focus on only two elementary price indices, namely we consider Jevons and Carli formulas. It was shown (Hardy et. al, 1934) that the difference between the Carli index and the Jevons index is bounded from below by the variance of the price relatives. In this paper we extend this result comparing expected values of these sample indices under the assumption that price relatives are described by geometric Brownian motion.

2 Unweighted Jevons and Carli indices

There are several elementary price indices in the literature (Von der Lippe, 2007).
In particular we have the following formulas

- the Carli price index (Carli, 1804)

$$P_C = \frac{1}{N}\sum_{i=1}^{N}\frac{p_i^t}{p_i^0}, \tag{1}$$

- the Jevons price index (Jevons, 1865)

$$P_J = \prod_{i=1}^{N}\left(\frac{p_i^t}{p_i^0}\right)^{\frac{1}{N}}, \tag{2}$$

where the time moment $\tau = 0$ we consider as the basis, N is the number of items observed at times 0 and t, p_i^τ denotes the price of the *i*-th item at time τ.

The Carli index is an arithmetic mean of price relatives (partial indexes), whereas the Jevons index is a geometric mean. As a consequence, these indices satisfy the classic inequality for arithmetic and geometric means

$$P_J \le P_C. \tag{3}$$

The difference between the Carli index and the Jevons index is bounded from below by the variance of the price relatives (Hardy at al., 1934):

$$P_C - P_J \ge D^2\left(\frac{p_i^t}{p_i^0}\right), \tag{4}$$

and thus the analogical inequality holds for their expected values. From the axiomatic price index theory the Jevons index seems to be better; it satisfies main tests (axioms), whereas the Carli index does not satisfy the time reversal test and circularity (Balk, 1995; Levell, 2015).

3 Comparison of expected values of sample indices

Let us treat price processes as stochastic ones and let the Carli index (1) and the Jevons index (2) be sample indices, where N denotes the sample size. Let us denote by P_C^0 and P_J^0 the following, unknown (a priori) values:

$$P_C^0 = \frac{1}{N}\sum_{i=1}^{N} E(\frac{p_i^t}{p_i^0}). \quad (5)$$

$$P_J^0 = \prod_{i=1}^{N} E(\frac{p_i^t}{p_i^0})^{\frac{1}{N}}. \quad (6)$$

In this section we are going to compare expected values of sample Jevons and Carli price indices in the continuous time stochastic model. We assume that prices are described by the geometric Brownian (Wiener) motion (also known as the exponential Brownian motion), i.e.

$$dp_i^t = \alpha_i p_i^t dt + \beta_i p_i^t dW_i^t, \quad (7)$$

where percentage drifts α_i and percentage volatilities β_i are constant, $\{W_i^t : 0 \le t < \infty\}$ are independent Wiener processes. The solution for the stochastic differential (7) is as follows (Oksendal, 2002)

$$p_i^t = p_i^0 \exp((\alpha_i - \frac{\beta_i^2}{2})t + \beta_i W_i^t), \quad (8)$$

and since we assume that all initial prices satisfy $p_i^0 = 1$ we obtain the following expected values of the price relatives P_i^t, where $i = 1,2,...,N$ (Jakubowski et al., 2003)

$$E(P_i^t) = E(\frac{p_i^t}{p_i^0}) = \exp(\alpha_i t). \quad (9)$$

Obviously, from (3) or (4) we know that $E(P_c) \ge E(P_J)$. Let us notice that it holds

$$P_J = \prod_{i=1}^{N} (P_i^t)^{\frac{1}{N}} = \prod_{i=1}^{N} \exp(\frac{\alpha_i - \beta_i^2/2}{N} t + \frac{\beta_i}{N} W_i^t), \quad (10)$$

or equivalently

$$P_J = \exp((\sum_{i=1}^{N} \frac{\alpha_i}{N} - \frac{1}{2}\sum_{i=1}^{N} (\frac{\beta_i}{N})^2)t + \sum_{i=1}^{N} \frac{\beta_i}{N} W_i^t) \exp(\frac{1}{2}(\sum_{i=1}^{N} (\frac{\beta_i}{N})^2 - \sum_{i=1}^{N} \frac{\beta_i^2}{N})t). \quad (11)$$

Let us denote by $vol(\beta_1, \beta_2, ..., \beta_N)$ a component connected with price volatilities, i.e.

$$vol(t, \beta_1, \beta_2, ..., \beta_N) = \exp(\frac{1}{2}(\sum_{i=1}^{N} (\frac{\beta_i}{N})^2 - \sum_{i=1}^{N} \frac{\beta_i^2}{N})t) = \exp(\frac{1-N}{2N^2}\sum_{i=1}^{N} \beta_i^2 t). \quad (12)$$

From (11) and (12) and under the assumption about independent Wiener processes we can write an expected value of the Jevons price index as follows

$$E(P_J) = vol(t, \beta_1, \beta_2, ..., \beta_N) \prod_{i=1}^{N} E[\exp((\frac{\alpha_i}{N} - \frac{1}{2}(\frac{\beta_i}{N})^2)t + \frac{\beta_i}{N} W_i^t). \quad (13)$$

In analogous way to (8) and (9) we obtain

$$E(P_J) = vol(t, \beta_1, \beta_2, ..., \beta_N) \prod_{i=1}^{N} \exp(\frac{\alpha_i}{N} t) = vol(t, \beta_1, \beta_2, ..., \beta_N) \prod_{i=1}^{N} (E(P_i^t))^{\frac{1}{N}}. \quad (14)$$

In the case of the Carli price index, from (1) and (9) we get

$$E(P_C) = \frac{1}{N} E(\sum_{i=1}^{N} \frac{p_i^t}{p_i^0}) = \frac{1}{N} \sum_{i=1}^{N} E(P_i^t) = \frac{1}{N} \sum_{i=1}^{N} \exp(\alpha_i t). \quad (15)$$

From (14) and (15) we have that $E(P_C) = P_C^0$ and $E(P_J) \neq P_J^0$, where

$$E(P_J - P_J^0) = (vol(t, \beta_1, \beta_2, ..., \beta_N) - 1) P_J^0. \quad (16)$$

Analogously to (3) we have

$$P_C^0 \geq P_J^0, \quad (17)$$

and thus

$$E(P_C - P_J) = P_C^0 - vol(t, \beta_1, \beta_2, ..., \beta_N) P_J^0 \geq P_J^0 (1 - vol(t, \beta_1, \beta_2, ..., \beta_N)). \quad (18)$$

Obviously, if price processes are deterministic, i.e. if $\beta_1 = \beta_2 = ... = \beta_N = 0$,we get trivial conclusion that $vol(t, \beta_1, \beta_2, ..., \beta_N) = 1$ and thus

$$E(P_C - P_J) = P_C - P_J = P_C^0 - P_J^0. \quad (19)$$

The main conclusion from the relation described in (18) is that the difference between expected values of the sample Carli index and the sample Jevons index depend on number of items, volatilities of price relatives and values of arithmetic and geometric means of expected values of sample price relatives. In particular, the inequality in (18) states that the higher the inflation is, the bigger differences between expected values of the Carli index and the Jevons index appear.

Remark

The estimation of variances of Carli and Jevons indices in the stochastic model would exceed the limited size of this paper and thus it is omitted. However, we calculate these statistics numerically in the simulation study (see Section 4).

4 Simulation study

Let us take into consideration a group of $N = 4$ items, the time horizon of observations $T = 1$ and the following parameters of price processes described in (7).

Case 1 (small volatilities)

$\alpha_1 = 0.02$, $\beta_1 = 0.05$, $\alpha_2 = 0.03$, $\beta_2 = 0.06$, $\alpha_3 = 0.05$, $\beta_3 = 0.015$,

$\alpha_4 = 0.06$, $\beta_4 = 0.01$.

Case 2 (medium volatilities)

$\alpha_1 = 0.02$, $\beta_1 = 0.25$, $\alpha_2 = 0.03$, $\beta_2 = 0.26$, $\alpha_3 = 0.05$, $\beta_3 = 0.15$,

$\alpha_4 = 0.06$, $\beta_4 = 0.1$.

Case 3 (big volatilities)

$\alpha_1 = 0.02$, $\beta_1 = 0.85$, $\alpha_2 = 0.03$, $\beta_2 = 0.76$, $\alpha_3 = 0.05$, $\beta_3 = 0.75$,

$\alpha_4 = 0.06$, $\beta_4 = 0.6$.

Without loss of generality we assume that $p_i^0 = 1$ for each $i \in \{1,2,3,4\}$. Some realizations of price relatives from Case 1 (for $t \in [0,1]$) are presented in Fig.1. Fig.2 presents $K = 10000$ sample realizations of each P_i^1 in Case 1. Basic statistics for generated K values of Jevons and Carli indices depending on the considered case are presented in Table 1 - 3.

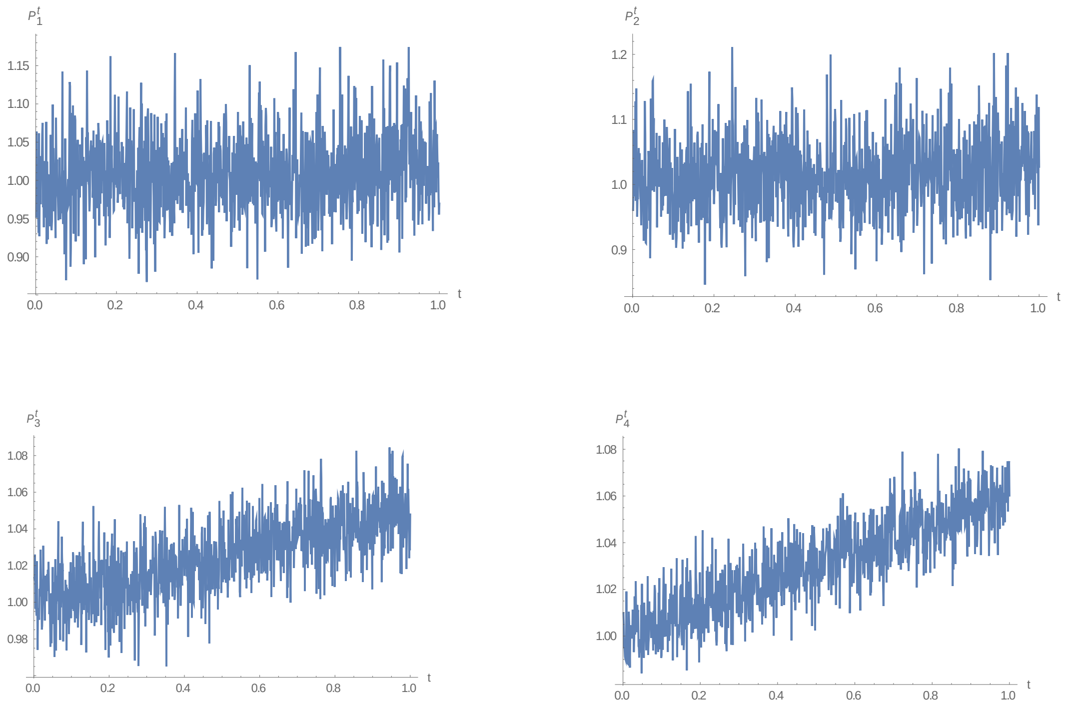

Fig. 1. Some realizations of price relatives processes for Case 1 and $t \in [0,1]$.

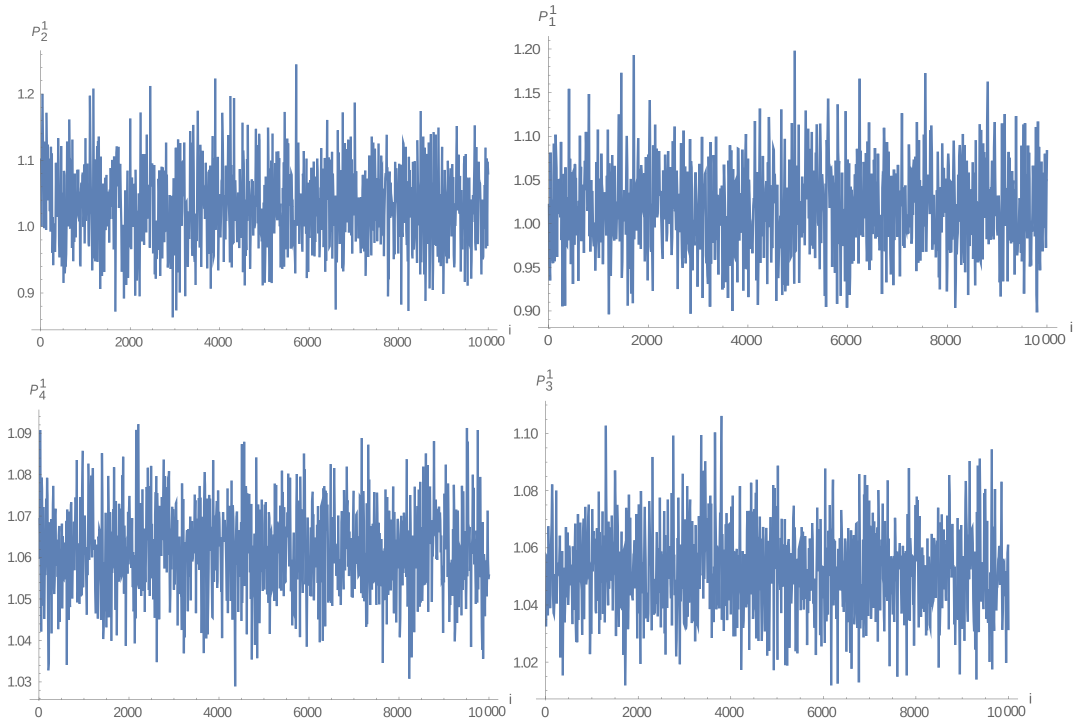

Fig. 2. Some realizations of price relatives processes for Case 1 and $t = T = 1$.

Table 1. Basic statistics for generated Jevons and Carli price indices (Case 1).

Basic statistics	Jevons index	Carli index
Mean	1.0411	1.0412
Standard Deviation	0.0204	0.0206
Volatility coefficient	0.0196	0.0198

Table 2. Basic statistics for generated Jevons and Carli price indices (Case 2).

Basic statistics	Jevons index	Carli index
Mean	1.0265	1.0415
Standard Deviation	0.1044	0.1059
Volatility coefficient	0.1017	0.1017

Table 3. Basic statistics for generated Jevons and Carli price indices (Case 3).

Basic statistics	Jevons index	Carli index
Mean	0.8435	1.0414
Standard Deviation	0.3232	0.4567
Volatility coefficient	0.3832	0.4385

Conclusions

There are several sources of the CPI bias including the elementary index bias (White, 1999). As it was mentioned, a choice of the elementary formula does matter in final inflation calculations. There has been a general trend in replacing the Carli index with the Jevons or the Dutot formulas and most of papers recommend the Jevons index rather than the Carli index. In the paper we show some similarities and differences in practical using of these indices. First of all, in our simulation study we observe that the expected (mean) value of generated values of the Jevons formula depends strongly on price volatilities whereas the mean value of generated values of the Carli index does not react on price fluctuations. In the case of strong price fluctuations, the differences between the expectations of Jevons and Carli price indices increase. In particular, we obtain the following $vol(t, \beta_1, \beta_2, ..., \beta_N)$ function values: 0.999 (Case 1 with small price volatilities), 0.984 (Case 2 with medium price volatilities) and 0.812 (Case 3 with high price volatilities). Thus, the differences between expected values of sample Jevons and Carli indices are the strongest in the Case 3. Moreover, we show analytically that the difference between expected values of the sample Carli index and the sample Jevons index depend on number of items, volatilities of price relatives and values of arithmetic and geometric means of expected values of sample price relatives. We also can observe (from the inequality in (18)) that the higher the inflation is, the bigger differences between expected values of sample Carli and Jevons indices appear. It is quite interesting that volatilities of these generated (in the simulation) indices, measured by their standard deviations and volatility coefficients, seem to be comparable although they still depend on price dispersions.

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References

Balk, M. (1995). Axiomatic Price Index Theory: A Survey. *International Statistical Review, 63*, 69-95.

Boskin, M. S. (1996). (Chair) *Advisory Commission to Study the Consumer Price Index. Towards a more accurate measure of the cost of living*. Washington DC.

Boskin, M. S., Delberger, E. R., Gordon, R. J., Griliches, Z., & Jorgenson, D. W. (1998). Consumer prices in the consumer price index and the cost of living. *Journal of Economic Perspectives*, *12*(1), 3-26.

Bureau of Labor Statistics (BLS). (2001). The experimental CPI using geometric means (CPI-U-XG). *http://www.bls.gov/cpi/cpigmtoc.htm.*

Carli, G. (1804). Del valore e della proporzione de'metalli monetati. In: *Scrittori Classici Italiani di Economia Politica*, *13*, 297-336.

Evans, B. (2012). *International comparison of the formula effect between the CPI and RPI. Office for National Statistics*. Newport.

Hardy, G. H., Littlewood, J. E, & Polya, G. (1934). Inequalities. *Cambridge: Cambridge University Press.*

Jakubowski, J., Palczewski, A., Rutkowski, M., & Stettner, Ł. (2003). *Matematyka finansowa. Instrumenty pochodne*. Warszawa: Wydawnictwa Naukowo-Techniczne.

Jevons, W. S. (1865). The variation of prices and the value of the currency since 1782. *J. Statist. Soc. Lond.*, *28*, 294-320.

Levell, P. (2015). Is the Carli index flawed?: assessing the case for new retail price index RPIJ. *J. R. Statist. Soc. A*, *178*(2), 303-336.

Oksendal, K. (2002). Stochastic Differential Equations: An Introduction with Applications. *Springer*.

Silver, H., & Heravi, S. (2007). Why elementary price index number formulas differ: Evidence on price dispersion. *Journal of Econometrics*, *140* (2007), 874-883.

White, A. G. (1999). Measurement Biases in Consumer Price Indexes. *International Statistical Review*, *67*(3), 301-325.

Von der Lippe, P. (2007). *Index Theory and Price Statistics*. Peter Lang.