The logistic regression in predicting spike occurrences in electricity prices

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Abstract

Electricity supply and demand are subject to weather conditions (temperature, wind speed, precipitation) as well as daily, weekly or yearly seasonality due to e.g. an intensity of business activities. These features have a significant impact on the market and price behaviour. As a result of the lack of storage capacity sharp movements of electricity prices are often observed. An ability of modelling and forecasting jumps and spikes plays the crucial role in risk management. In the paper, the logistic regression is employed to predict spike occurrences. We investigate the impact of fundamental variables such as demand, weather and seasonal factors, on spikes occurrences. The point and interval theoretical probabilities are calculated. The classification accuracy is assessed by means of the *sensitivity, specificity, accuracy* and *AUC* measures. In our research we detect spikes using a quantile technique and a Bayesian *DEJD* model. We state that the logistic regression is a quite good tool to forecast moments of a spike occurrence. The logistic regression model is a well-known specification which seems to be reasonable tool of spike prediction.

Keywords: *electricity price, jumps, spike prediction, logistic regression, DEJD*

JEL Classification: C22, C53, Q02, Q41, Q47

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1 Introduction

Energy markets are different from other commodity and financial markets due to the difficulty in storing large quantities of electricity. At the same time, power system stability requires a constant balance between production and consumption. Electricity supply and demand are subject to weather conditions (temperature, wind speed, precipitation, solar radiation), daily, weekly or yearly seasonality due to e.g. an intensity of business activities (working hours, peak hours, weekdays, holidays, near holidays). These features have a significant impact on the market and price behaviour. They might result in extreme spot price volatility and may in consequence bring the existence of sharp price movements, i.e. spikes and jumps. An ability of modelling and forecasting them plays the crucial role in electricity price forecasting and risk management.

In comparison to the extensive literature devoted to forecasting electricity prices, relatively less attention is paid to forecasting price spikes (see e.g. Weron, 2014). One of the

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first work about forecasting spikes is Christensen's et al. (2012) paper, where the ACH model and logit model are employed. Eichler et al. (2012, 2014) compare the ACH and the logit model indicating the advantage of the latter.

The main objective of the article is the forecasting of electricity price spikes by means of the logistic regression. We investigate the impact of fundamental variables such as demand, weather and seasonal factors on spike occurrences, and determine point forecasts and 95% prediction intervals for a spike occurrence probability. The analysis is conducted for a standard level of a cut point (0.5) as well as a cut point calculated on the basis of the sensitivity-specificity plot. The quality of the classification is examined by means of the *accuracy*, *sensitivity*, *specificity* and the *AUC* measures.

2 Electricity spot prices and spikes

The term of a spike is in common use, but there is no unique and broadly accepted definition of it. In a certain simplification sudden and sharp movements of values in time series may be treated as spikes. While analysing a given time series we can easily discern the values which are spikes 'for sure' on account of their distinct outlying with respect to other observations. However, at the same time we do not know how to classify the remaining data points at hand – that is where to put the border line between observations which should and the ones which should not be classified as spikes. There are various methods of detecting spikes. Values surpassing a fixed threshold (see e.g. Christensen et al., 2012) or a threshold identified by some method or model are frequently classified as spikes. Actually, different methods lead to different moments of spike occurrences (see e.g. Janczura et al., 2013).

The research is based on the series of hourly electricity prices on Nord Pool market. Nord Pool is a leading power market and the largest one in Europe. About 380 members from 20 countries are active at Nord Pool. We focus on a day-ahead market (*Elspot*) on which 98% of electricity volume handled by the Nord Pool is traded. We model the hourly system price (EUR/MWh) which is an unconstrained market clearing reference price. Most standard financial contracts traded in the Nordic region use the system price as the reference price. At 12.00 a.m. all purchase and sell orders are aggregated into two curves for each delivery hour, and then the system prices are calculated for each delivery day-ahead hour.

In our research, the period of the analysis ranges from 2014/12/29 to 2017/07/02 and is divided into an in-sample (2014/12/29–2016/09/11) and an out-of-sample (2016/09/12–2017/07/02) period. The series of spot electricity prices is very volatile and 'spiky'. The highest prices are more than four times greater than the median price over analysed period.

Moreover, the prices are subject to intra-daily (24-hourly) and intra-weekly seasonality (see Fig. 1): higher electricity prices appear during on-peak hours, and lower prices during off-peak hours as well as during weekends.

Notably due to the lack of storage capacity sharp movements of prices are very frequent and violent on energy market. The power production, and hence the electricity price, is related to weather conditions. There are some seasonality patterns, especially in hydropower, but also in wind power production. The power consumption is intensified during business hours (on-peak hours) and also depends on the weather conditions (temperature, seasons). All of these factors may impact on prices and their abnormal sharp movements i.e. spike occurrences.

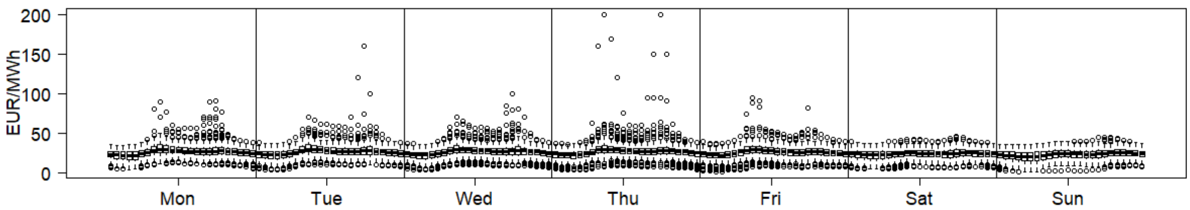

Fig. 1. The boxplots for electricity prices for each hour of a week: 2014/12/29–2017/07/02.

3 Applied methods and empirical results

In our research we adopt two methods of spike detection. The first technique identifies 2.5% of the highest values as spikes – likewise the variable price threshold method in (Janczura et al., 2013) or the quantile analysis in (Kostrzewska et al., 2016). We refer to it as the *QUA* technique. The second one is based on the Bayesian *DEJD* model considered by Kostrzewski (2015). We concentrate on the upwards spikes. Before applying the spike detection techniques we pre-process the series of the spot electricity prices as follows. Firstly, we get rid of the long-term seasonality component (LTSC, seasonal patterns in seasons, months etc.) by means of the Hodrick-Prescott filter[3], and thereafter, the short-term seasonal component (STSC, intra-weekly and intra-daily seasonal patterns) by means of the filter based on medians of the prices for each hour of a week[4]. Ultimately, we identify spikes in the remaining irregular component i.e. after applying the HP filter and the median STSC technique.

[3] Weron and Zator (2015) proposed using the HP filter for identifying the LTSC in electricity spot prices, which is less computationally complex and gives similar results to the wavelet technique.

[4] See (Janczura et al., 2013) for details about the filter based on the means. We apply more robust medians instead of the means.

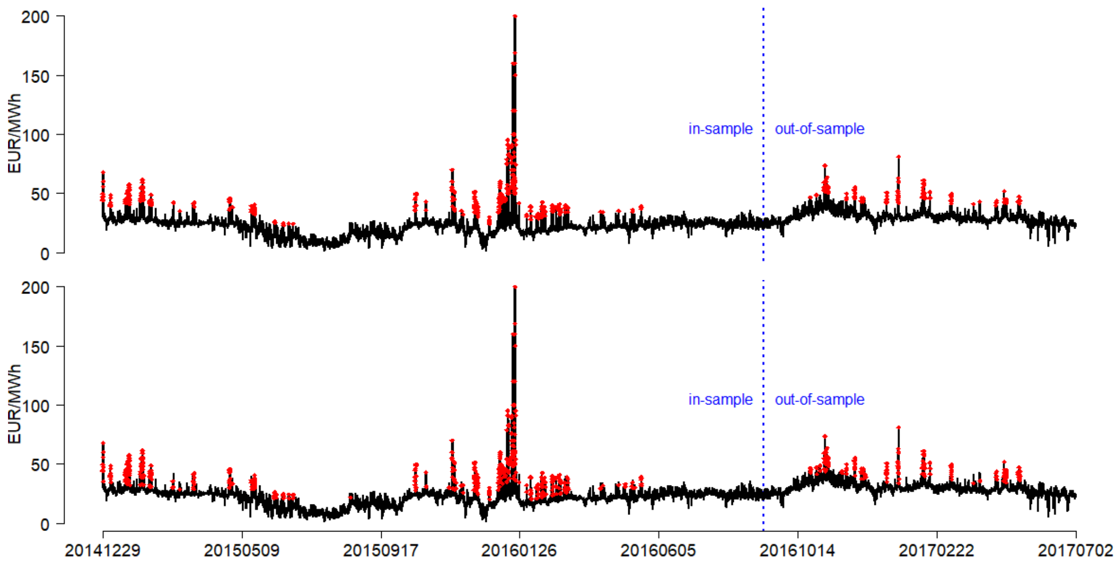

Fig. 2. The series of the hourly spot electricity prices with spikes (red) identified by the *QUA* (top) and the *DEJD* technique (bottom) over the period 2014/12/29–2017/07/02.

The electricity price spikes detected by means of the quantile technique *QUA* (2.5%) and the *DEJD* model (3.04%) are presented in Fig. 2. Not all spikes identified by the *QUA* are spikes according to the *DEJD*. Applying the HP and median filters enables to detect spikes in electricity prices regardless of the seasonal nature of prices. The blue dotted line indicates the border between in-sample and out-of-sample periods used in the paper.

Next, we employ the logistic regression in order to forecast moments of the spot electricity price spike occurrences. The dependent variable distinguishes between spikes (1's) identified by means of the one of the mentioned techniques and 'ordinary' values (0's). The in-sample period covers the period from 2014/12/29 to 2016/09/11 (14,952 hourly observations). We apply the extending window algorithm by 24 hours at a time with model re-estimation. Initially, we estimate the model and calculate the first day-ahead forecast covering 24 hours of a next day. Secondly, we extend the in-sample data set by 24 hours, estimate the model and calculate the next 24-hourly day-ahead forecast, and so on. The forecast is conducted in the out-of-sample period (2016/09/12–2017/07/02, 7,056 hourly observations), the forecast horizon covers 294 days and 294 logistic models are estimated.

In the logistic regression model, following exogenous variables are considered[5]: six dummy variables indicating days in a week (except Saturday – a reference day), wintertime (*winter*, dummy variable), on-peak hours (#8–#20) (dummy variable *peak*), electricity

[5] The variables *cons*, *minprice*, *lagprice*, *wind* and *hydro* are transformed so that the LTSC and STSC or only the LTSC is filtered out.

consumption forecasts (*cons*, MWh), wind power forecasts (*wind*, MWh), reservoir water levels (*hydro*, GWh), the minimum of the day-before hourly prices (*minp*, EUR/MWh), lagged by 48 hour prices (*lagp*, EUR/MWh), lagged by 48 hours failures of power plants (*fail*). The stepwise method is applied to select a final set of exogenous variables. Table 1 shows how frequently each variable occurs in the logistic regression models. The value '1' means that the variable is present in each of the 294 models, '0' – in none of them.

For each spike detection technique the variables such as: *Mon–Fri*, *peak*, *cons*, *wind* and *minp* explain the moments of spike occurrences for all logistic regression models. The results depict that only the dummy variable *Sun* does not explain the spike occurrences (with Saturdays as the reference). That means spikes on Saturdays behave almost the same as on Sundays, but differently than on the other days. The variable *winter* is present in 95.6% of 294 models (*QUA*) or in all models (*DEJD*), thus a distinction between winter and summer seasons is important in the majority of the models. The same conclusion can be made for the lagged by 48 hours failures of power plants (*fail*). The reservoir water level (*hydro*) is not so important and stays only in 12.9% (*QUA*) or 70.7% (*DEJD*) of 294 models. On the other hand, lagged by 48 hour prices (*lagp*) are important for all models in the case of the spike identification by the *QUA* and none of the models in the case of the *DEJD*.

Table 1. Frequency of an occurrence of each variable among 294 logistic regression models – spikes are identified by the quantile analysis (*QUA*) or the *DEJD* model.

	Mon	***Tue***	***Wed***	***Thu***	***Fri***	***Sun***	***peak***	***winter***	***cons***	***wind***	***hydro***	***minp***	***lagp***	***fail***
QUA	1	1	1	1	1	0	1	0.956	1	1	0.129	1	1	1
DEJD	1	1	1	1	1	0	1	1	1	1	0.707	1	0	0.980

We calculate the theoretical (point) probability and 95% prediction intervals for a probability of a spike occurrence for each hour (see Fig. 4-5). As mentioned before, the analysis is conducted for the standard level of a cut point (0.5) and the cut point calculated on the basis of the sensitivity-specificity plot (see Hilbe, 2016), named as the s-s cut point later in the paper. Fig. 3 presents the values of the s-s cut point for each of 294 estimated models. The values vary for individual models, and in the *QUA* approach are lesser than in the *DEJD*. In addition, we can compare the s-s cut point with the one calculated as the mean of the predicted values, which is often used in practice if the dependent variable has substantially more or less 1's than 0's as mentioned by Hilbe (2016). The average value of the forecasted probabilities of a spike occurrence is equal to 0.030 (*QUA*) or 0.037 (*DEJD*), which indicates that the s-s cut point based on the sensitivity-specificity plot may be more appropriate than the standard cut point equal to 0.5.

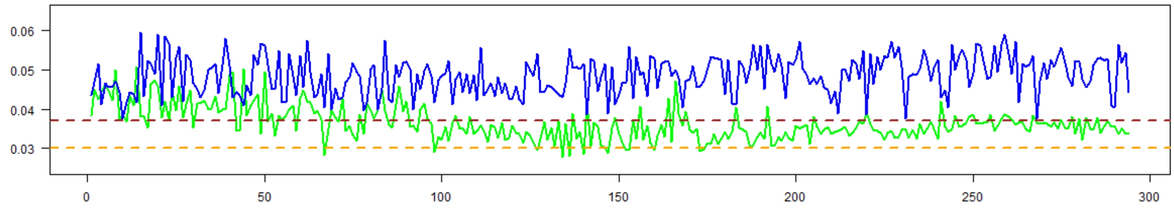

Fig. 3. The values of the s-s cut point (green – the *QUA*, blue – the *DEJD*) for each of 294 logistic regression models and the average predictive values of probability of spike occurrence (orange dashed line – the *QUA*, brown dashed line – the *DEJD*).

The quality of the classification is assessed by means of the *sensitivity*, *specificity*, *accuracy* and *AUC* measures over the in-sample and out-of-sample periods, separately, in the case of the cut point equal to 0.5 and the s-s cut point (see Table 2). The *sensitivity* is a percentage of correctly classified spikes among all spikes that occurred (i.e. identified by the method). The *specificity* is a percentage of correctly classified 'ordinary' prices among all 'ordinary' prices that occurred. The *accuracy* is a percentage of correctly classified spikes and 'ordinary' prices together. The *AUC* is an area under the receiver operator characteristic (*ROC*) curve.

Table 2. The in-sample and out-of-sample assessment of the quality of the classification: the average *sensitivity*, *specificity*, *accuracy* and *AUC* measures for 294 logistic regression models with the cut point equal to 0.5 and the s-s cut point – the *QUA* or *DEJD* approach.

		In-sample				**Out-of-sample**			
	Cut point:	***Sens***	***Spec***	***Accur***	***AUC***	***Sens***	***Spec***	***Accur***	***AUC***
QUA	0.5	55.5	**99.5**	**98.3**	77.5	57.4	**99.0**	**98.2**	78.2
	s-s	**95.5**	93.5	93.5	**94.5**	**96.3**	90.7	90.8	**93.5**
DEJD	0.5	48.7	**99.4**	**97.8**	74.0	56.2	**99.0**	**97.9**	77.6
	s-s	**90.8**	92.4	92.4	**91.6**	**94.6**	89.5	89.7	**92.1**

On the basis of the results reported in Table 2 we conclude that the assessment of the models by means of the *sensitivity* and *AUC* measures is higher when using the s-s cut point than the cut point equal to 0.5, for both the *QUA* as well as the *DEJD* technique. This is due to the percentage of spikes identified by the method (2.5% – *QUA*, 3.04% – *DEJD*) is closer to the mean optimal cut point (0.036 – *QUA*, 0.049 – *DEJD*) than to the value 0.5. However, we note the lower values of the *specificity* and *accuracy* measures when the s-s cut point is employed, but still suitably high. This may be caused by the higher number of 'false alarms' (incorrectly predicted spikes) when using the lower cut point.

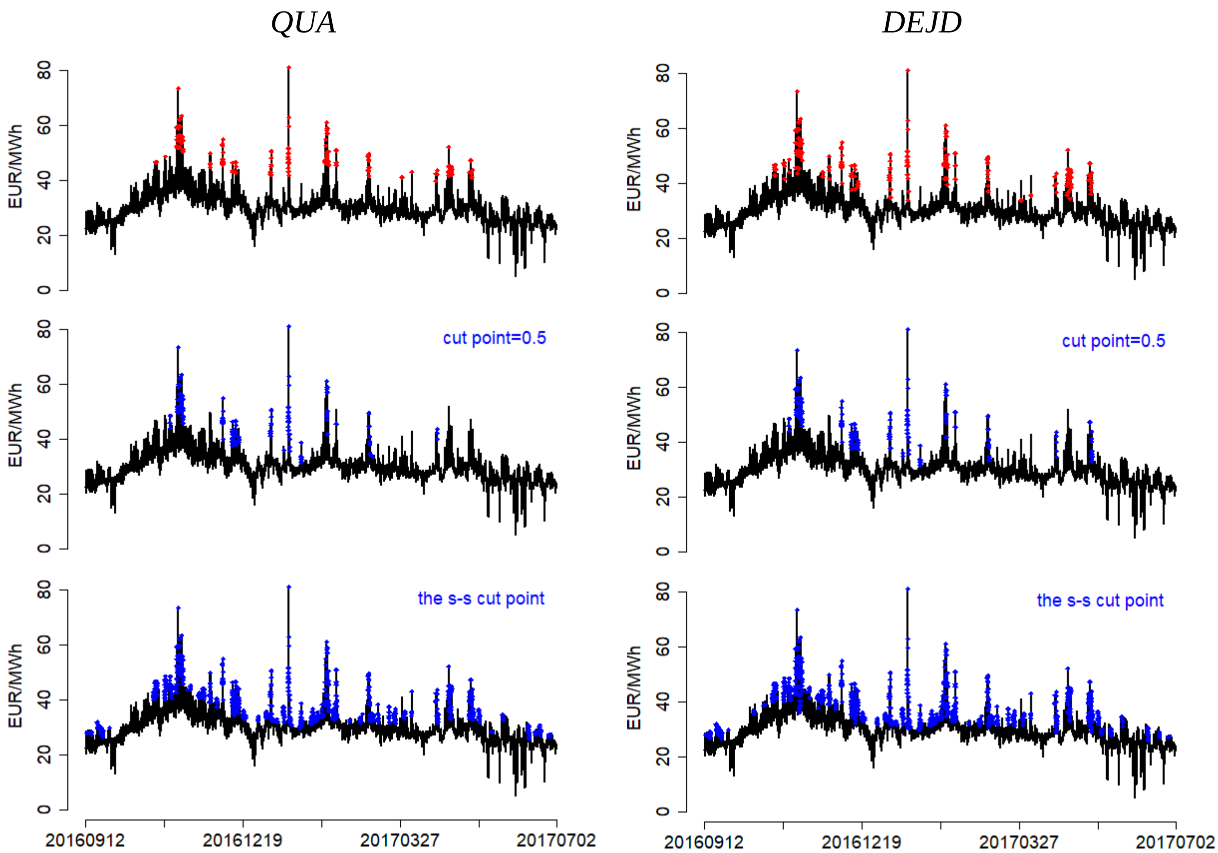

Fig. 4. The electricity price series over the out-of-sample period with moments of spike occurrences (red) detected by means of the *QUA* (top left) or the *DEJD* (top right) technique and the forecasted spikes (blue) with the 0.5 cut point (middle left and right) or the s-s cut point (bottom left and right).

Fig. 4 presents the moments of spike occurrences identified by means of the quantile technique and two variants of spike forecasts: using the cut point equal to 0.5 or the s-s cut points calculated for 294 models. The visual analysis indicates that the spike forecast with the s-s cut point is more reasonable than with the cut point equal to 0.5, although in the first case there are more 'false alarms' than in the later one. Such forecasts might be interesting for retailers who prefer to hedge on the financial market due to the 'false alarm' than set loss down. The conclusions coincide with those drawn from the analysis of the quality of the classification made before.

In the research, we also determine the 95% prediction intervals for the probability of the spike occurrence for each hour (see Fig. 5). The prediction intervals enable to define forecast rules in a more or less restrictive way. In the more restrictive approach, the spike occurrence is forecasted if the entire prediction interval is above the cut point. In that case the number of 'false alarms' is reduced in comparison to the forecasts based on the point theoretical probabilities. On the other hand, in the less restrictive approach, the spike occurrence is forecasted when only the upper bound of the prediction interval is above the cut point. We adopt both approaches and calculate the *sensitivity*, *specificity*, *accuracy* and *AUC* measures with different cut points in order to compare the results with those in the case of the point forecasts of spike occurrences (see Table 3).

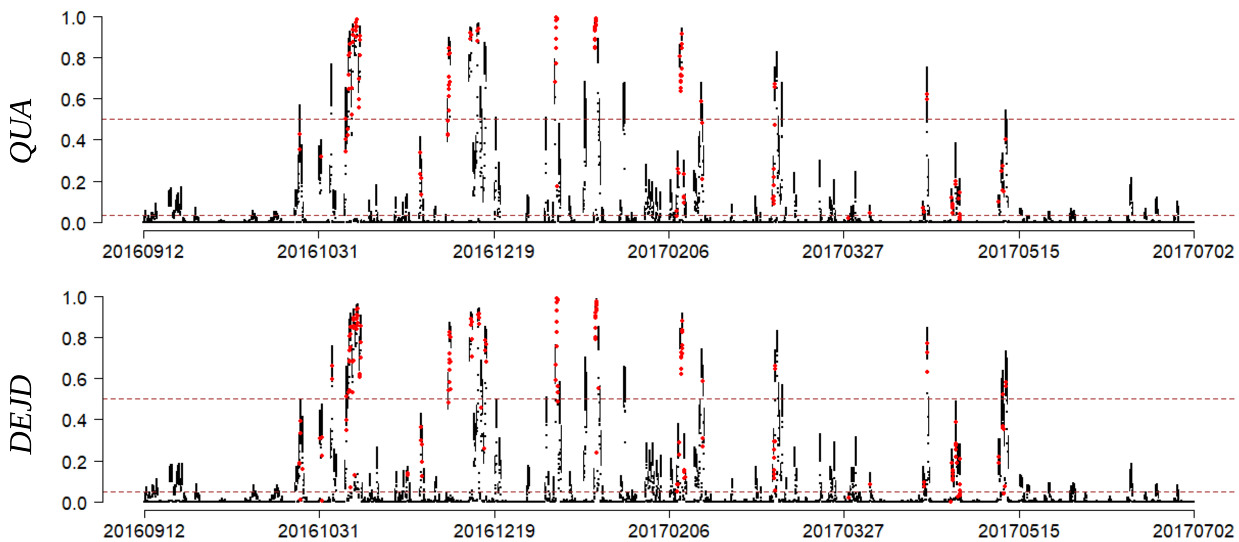

Fig. 5. The 95% prediction intervals of spike occurrences (black) constructed on the basis of the logistic regression. Horizontal dashed lines indicate the average s-s and 0.5 cut point. The theoretical probabilities for spikes detected by means of the *QUA* (top) or the *DEJD* technique (bottom) are depicted in red. If the red points are above the dashed line the spikes predicted by means of the logistic model coincide with those detected by means of the *QUA* or *DEJD.*

The results in Table 3 indicate that the highest values of the *sensitivity* and *AUC* measures are observed for the less restrictive rule of spike forecasts (upper bound), next for the forecasts based on the point theoretical probability and the lowest values – for the more restrictive approach (entire interval). On the other hand, the highest values of the *specificity* and *accuracy* measures are noted for the more restrictive approach, then for the point forecasts and for the less restrictive approach – the lowest. That is, the less restrictive rule of spike forecasting brings a higher percentage of correctly predicted spikes in combination with a higher percentage of ‘false alarms’, and conversely. It is worth mentioning that for all approaches the AUC measure is above 90% with the s-s cut point and above 75% with the cut point equal to 0.5. These results indicate that the logistic regression model is a good tool to classify and forecast spike occurrences in the spot electricity prices.

Table 3. Out-of-sample assessment of the quality of the classification: the average *sensitivity*, *specificity*, *accuracy* and *AUC* measures for 294 logistic regression models with the cut point equal to 0.5 and the average s-s cut point – the results for the more restrictive approach (Upper), point forecast (Point) or less restrictive approach (Entire).

	QUA						***DEJD***					
Cut	**Upper**		**Point**		**Entire**		**Upper**		**Point**		**Entire**	
point:	**s-s**	**0.5**	**s-s**	**0.5**	**s-s**	**0.5**	**s-s**	**0.5**	**s-s**	**0.5**	**s-s**	**0.5**
Sens	98.5	64.7	96.3	57.4	91.2	52.2	96.2	58.9	94.6	56.2	90.3	48.1
Spec	88.9	98.6	90.7	99.0	93.4	99.4	87.3	98.6	89.5	99.0	92.4	99.4
Accur	89.1	97.9	90.8	98.2	93.3	98.4	87.5	97.5	89.7	97.9	92.3	98.1
AUC	93.7	81.7	93.5	78.2	92.3	75.8	91.8	78.7	92.1	77.6	91.3	73.8

Conclusions

In the research, the time series of electricity prices is an imbalanced dataset – there are more ‘ordinary’ values than the spikes. In consequence, adoption the standard cut point equal to 0.5 leads to worse results than the s-s cut point. The value of the s-s cut point is similar to the percentage of observations identified as spikes by means of the quantile (*QUA*) as well as the *DEJD* technique. The more restrictive approach based on 95% prediction intervals forecasts correctly less ‘real’ (i.e. identified by the method) spikes and less ‘false alarms’ in comparison with the forecast based on the point theoretical probability of spike occurrences. On the other hand, the less restrictive approach predicts correctly more spikes and more ‘false alarms’ than the point forecast method.

The exogenous variables have strong impact on the spikes prediction. We consider two variables corresponding to renewable energy sources – the wind power forecasts and the reservoir water levels. The wind power forecasts are important for all models. However, the reservoir water levels are employed only in 38 (in the case of the *QUA* technique) and 208 (the *DEJD*) out of 294 models. It might be explained by lower volatility of the latter variable which is constant during each week. Moreover, we would like to note that the failures of power plants variable is employed for almost all logistic regression models.

The logistic regression model is a well-known specification which seems to be reasonable tool of spike prediction. In our future research we are going to employ and compare other methods of spike detection and prediction.

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