Application of the AG Mean Index for CPI Substitution Bias Reduction

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Abstract

The Consumer Price Index (CPI) approximates changes in the costs of household consumption assuming the constant utility (COLI, Cost of Living Index). In practice, the Laspeyres price index is used to measure the CPI despite the fact that many economists consider the superlative indices to be the best approximation of COLI. The Fisher index is one of the superlative indices and, additionally, it satisfies most of tests from the axiomatic price index theory. Nevertheless, the Fisher price index makes use of current-period expenditure data and its usefulness in CPI measurement is limited. Lent and Dorfman (2009) show that a weighted average of the arithmetic and geometric Laspeyres indices (the so-called AG mean index) used as a proxy for the Fisher formula can provide a simple alternative to the Lloyd-Moulton index. To use the AG mean index in practice we have to approximate the right parameter being in the index's body. Theoretically, the parameter should not change rapidly over time since it denotes the elasticity of substitution. In the paper we apply the AG mean index for the Fisher index approximation using CPI data from the United Kingdom and Bulgaria. The main aim of the paper is to examine fluctuations in the estimated parameter and its dependence on the level of data aggregation.

Keywords: *CPI, COLI, the Fisher index, the Laspeyres index, the AG Mean index*
JEL Classification: C43
DOI: 10.14659/SEMF.2018.01.41

1 Introduction

The Consumer Price Index (CPI) is commonly used as a basic measure of inflation. The index approximates changes in the costs of household consumption assuming the constant utility (COLI, Cost of Living Index). In practice, the Laspeyres price index is used to measure the CPI (White, 1999; Clements and Izan, 1987), although the Laspeyres formula does not take into account changes in the structure of consumption, which occur as a result of price changes in the given time interval. Many economists consider the superlative indices (like the Fisher index or the Törnqvist index) to be the best approximation of COLI (Von der Lippe, 2007). The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. However, there are some other ways to reduce that bias, like using the Lloyd–Moulton price index (see Lloyd, 1975; Moulton, 1996; Shapiro and

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Wilcox, 1997; Białek, 2014), the AG Mean index (Lent and Dorfman, 2009; Białek, 2017) or the Lowe and Young indices (Armknetch and Silver, 2012).

In the paper we focus on application of the AG mean index for CPI substitution bias reduction. To use the AG mean index in practice one needs to approximate the right parameter being in the index's body. Theoretically, the parameter should not change rapidly over time since it denotes the elasticity of substitution. In the paper we apply the AG mean index for the Fisher index approximation using CPI data from the United Kingdom and Bulgaria[3]. The main aim of the paper is to examine fluctuations in the estimated parameter and its dependence on the level of data aggregation.

2 Cost of Living Index (COLI) vs Consumer Price Index (CPI)

Let $E(P,\bar{u}) = \min_{Q}\{P^T Q | U(Q) \geq \bar{u}\}$ be the expenditure function of a representative consumer which is dual to the utility function $U(Q)$. In other words, it is the minimum expenditure necessary to achieve a reference level of utility $\bar{u}$ at the vector of prices P. Then, the Konüs cost of living price index is defined as

$$P_K = \frac{E(P^t,\bar{u})}{E(P^s,\bar{u})}, \tag{1}$$

where t denotes the current period, s denotes the base period, and in general, the vector of N considered prices at any moment τ is given by $P^\tau = [p_1^\tau, p_2^\tau, ..., p_N^\tau]^T$. I_K is the true cost of living index in which the commodity Q changes as the vector of prices facing the consumer changes. The CPI, in contrast, measures the change in the cost of purchasing a fixed basket of goods at a fixed sample of outlets over a time interval, *i.e.* $Q^s = [q_1^s, q_2^s, ..., q_N^s]^T = Q^t$. The CPI is a Laspeyres-type index being the weighted arithmetic mean of price relatives

$$P_{La} = \frac{\sum_{i=1}^{N} q_i^s p_i^t}{\sum_{i=1}^{N} q_i^s p_i^s} = \sum_{i=1}^{N} w_i^s \frac{p_i^t}{p_i^s}, \tag{2}$$

so we assume here the constant consumption vector on the base period level. It can be shown (Diewert, 1993) that

[3] Currently there are no differences between the CPI and HICP (Harmonized Index of Consumer Prices) indices in the case of these countries. In our research we use HICP data from Eurostat from COICOP-4 digit and COICOP-3 digit levels of aggregation.

$$P_K = \frac{E(P^t, U(Q^s))}{E(P^s, U(Q^s))} \leq P_{La}, \tag{3}$$

so $P_{La} - P_K$ is the extent of the commodity substitution bias.

3 Proxies for the Fisher price index

In the so-called economic price index approach many authors use superlative price indices to approximate the P_K index (White, 1999). The most popular superlative index is the Fisher index P_F (Fisher, 1922), which can be written as the geometric mean of the Laspeyres (P_{La}) and Paasche (P_{Pa}) price indices, i.e.

$$P_F = \sqrt{P_{La} P_{Pa}}. \tag{4}$$

where

$$P_{Pa} = \frac{\sum_{i=1}^{N} q_i^t p_i^t}{\sum_{i=1}^{N} q_i^t p_i^s}. \tag{5}$$

Nevertheless, the Fisher price index makes use of current-period expenditure data and its usefulness in CPI measurement is limited. Using a Constant Elasticity of Substitution (CES) framework, a superlative Fisher price index can be approximated once the elasticity of substitution (σ) is estimated. The Lloyd–Moulton price index (Lloyd, 1975; Moulton, 1996; Shapiro and Wilcox, 1997) does not make use of current-period expenditure data, and it follows the formula

$$P_{LM}(\sigma) = \{[\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{1-\sigma}]^{\frac{1}{1-\sigma}}, \tag{6}$$

where σ is some real parameter and w_i^s denotes the expenditure share of commodity i in the base period s. Unfortunately, there is no analytical solution for the equation $P_F = P_{LM}(\sigma)$ with respect to σ and thus this parameter can be calculated only numerically (Feenstra and Reinsdorf, 2007; Biggeri and Ferrari, 2010; Greenlees, 2011; Armknecht and Silver, 2012; Białek, 2017).

Lent and Dorfman (2009) prove that a weighted average of the Laspeyres index and the geometric Laspeyres index can approximate the Lloyd-Moulton index and thus it also approximates the superlative target index. In particular, the above-mentioned weighted average (called the AG Mean index) provides a close approximation to the Fisher price index, namely

$$P_F \approx P_{AG} = \sigma \prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^s}\right)^{w_i^s} + (1-\sigma) \sum_{i=1}^{N} w_i^s \left(\frac{p_i^t}{p_i^s}\right). \quad (7)$$

Solving the equation $P_F = P_{AG}$ with respect to σ we obtain (Armknecht and Silver, 2012)

$$\hat{\sigma} = \frac{P_F - P_{La}}{P_{GLa} - P_{La}}, \quad (8)$$

where P_{GLa} denotes the geometric Laspeyres price index (Von der Lippe, 2007)

$$P_{GLa} = \prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^s}\right)^{w_i^s}. \quad (9)$$

Since the parameter should not change rapidly over time, we can estimate it using historical data. Apparently, we should have a good tool for approximation of the current value of the Fisher price index. In the empirical study we verify the utility of the AG Mean index in CPI substitution bias reduction using CPI (HICP) data from the United Kingdom and Bulgaria.

4 Empirical study

In the following section we apply the AG Mean index for the Fisher index approximation using CPI data from the United Kingdom and Bulgaria. Currently there are no differences between the CPI and HICP (Harmonized Index of Consumer Prices) indices in the case of these countries. Thus, we use yearly data from Eurostat from COICOP-4 digit and COICOP-3 digit levels of aggregation, and we calculate main price indices and also the parameter of the elasticity of substitution (8) for years 2011-2016.

Table 1 and Table 2 present results for Bulgaria, whereas Tab. 3 and Tab. 4 concentrate on the United Kingdom.

Table 1. Price indices and the estimated parameter of the elasticity of substitution.

Index	**Bulgaria (COICOP 3-digit)**					
formula	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
P_{La}	1.03645	1.02033	1.00396	0.98295	0.98841	0.98592
P_{GLa}	1.03546	1.01960	1.00365	0.98256	0.98738	0.98540
P_{Pa}	1.03393	1.02381	1.00392	0.98389	0.99047	0.98730
P_F	1.03519	1.02207	1.00394	0.98342	0.98944	0.98661
$\hat{\sigma}$	1.26854	-2.39129	0.06734	-1.22779	-0.99247	-1.32732

Notes: Bulgaria, years: 2011-2016, data from COICOP 3-digit level of aggregation.

Table 2. Price indices and the estimated parameter of the elasticity of substitution.

Index formula	Bulgaria (COICOP 4-digit)					
	2011	**2012**	**2013**	**2014**	**2015**	**2016**
P_{La}	1.04385	1.02504	1.00469	0.98087	0.98481	0.98419
P_{GLa}	1.04205	1.02411	1.00419	0.98019	0.98304	0.98294
P_{Pa}	1.03839	1.02772	1.00536	0.98119	0.98797	0.98581
P_F	1.04111	1.02638	1.00503	0.98103	0.98639	0.98500
$\hat{\sigma}$	1.51943	-1.44693	-0.66349	-0.23119	-0.89077	-0.65484

Notes: Bulgaria, years: 2011-2016, data from COICOP 4-digit level of aggregation.

Table 3. Price indices and the estimated parameter of the elasticity of substitution.

Index formula	United Kingdom (COICOP 3-digit)					
	2011	**2012**	**2013**	**2014**	**2015**	**2016**
P_{La}	1.04425	1.02648	1.02237	1.01129	0.99802	1.00485
P_{GLa}	1.04339	1.02609	1.02207	1.01104	0.99759	1.00456
P_{Pa}	1.04417	1.02732	1.02227	1.01222	0.99839	1.00581
P_F	1.04421	1.02690	1.02232	1.01175	0.99821	1.00533
$\hat{\sigma}$	0.04953	-1.07063	0.17058	-1.90674	-0.42942	-1.67218

Notes: United Kingdom, years: 2011-2016, data from COICOP 3-digit level of aggregation.

Fig. 1 compares $\hat{\sigma}$ estimates calculated for Bulgaria and the United Kingdom for each year of observation and depending on the level of aggregation (COICOP). Our results show that the application of the AG Mean index for CPI substitution bias reduction (or, equivalently, for the Fisher price index approximation) is limited in practice.

Table 4. Price indices and the estimated parameter of the elasticity of substitution.

Index formula	United Kingdom (COICOP 4-digit)					
	2011	**2012**	**2013**	**2014**	**2015**	**2016**
P_{La}	1.04591	1.02164	1.02001	1.00813	0.98914	0.99841
P_{GLa}	1.04473	1.02085	1.01954	1.00777	0.98817	0.99797
P_{Pa}	1.04557	1.02311	1.01990	1.00858	0.98978	0.99926
P_F	1.04574	1.02237	1.01996	1.00835	0.98946	0.99884
$\hat{\sigma}$	0.14339	-0.93094	0.10857	-0.63530	-0.33362	-0.98098

Notes: United Kingdom, years: 2011-2016, data from COICOP 4-digit level of aggregation.

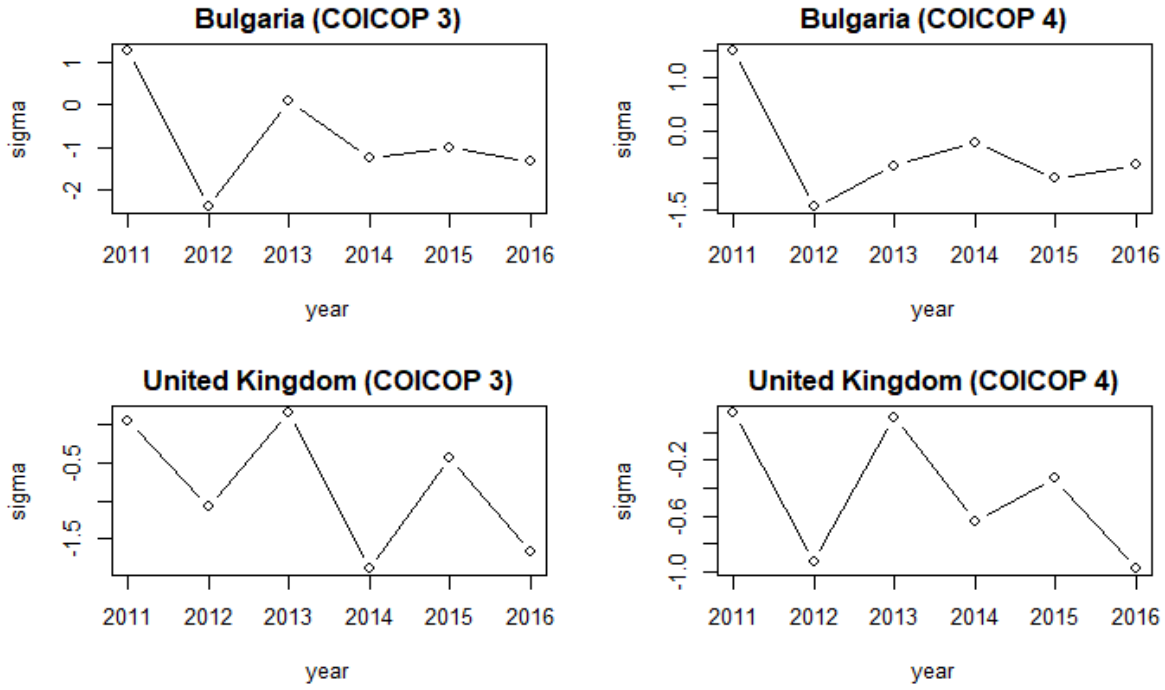

Fig. 1. Estimates $\hat{\sigma}$ for Bulgaria and the United Kingdom for years 2011-2016.

Conclusions

In our opinion the usefulness of the AG Mean index as a proxy for the ideal Fisher price index is very limited in practice. From a theoretical point of view, the AG Mean index seems to be a great alternative to the Laspeyres price index in inflation (CPI) measurement, since it approximates the Fisher index (considered the best approximation of Cost of Living Index) and it does not need the current-period expenditure data. Nevertheless, estimation of the parameter σ, which reflects the elasticity of substitution, has some practical drawbacks.

Firstly, its estimator given by (8) do need the current-period expenditure data. Obviously, we could change the method of estimation of σ using some econometric approach or numerical methods for the Lloyd-Moulton index, but the usefulness of any estimations is still bounded since this parameter strongly fluctuates in time (see Fig. 1). Thus, the computed value of this parameter for the year $t-1$ as a rule is completely different from its value calculated for the current year t (see Fig. 1 and Table 1 – 4). Secondly, the considered parameter seems to strongly depend on the level of aggregation (compare Table 1 with Table 2 and Table 3 with Table 4). The main conclusion from these observations is that within the framework considered in this paper it is impossible to provide a single, general value of the parameter σ or to propose some narrow interval of its possible values. Thus, the application of the AG Mean index for CPI substitution bias reduction is doubtful in practice.

Acknowledgements

Authors of the paper would like to thank the National Science Centre in Poland for financing this publication (grant no. 2017/25/B/HS4/00387).

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