Verification of usability of the price cobweb model in current market conditions

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Abstract

Specific relations between the price and supply in the market of vegetable products are determined by many causes of environmental and biological nature as well as by the market situation. It results in a specified alternation of these values and affects income implemented by producers in this market. The basis of this analysis is the law of supply (the price rise results in the rise in the supply volume while the price fall – in the fall of the supply) and its flexibility in relation to the price. The objective of these studies is to verify the assumptions of the cobweb model (used originally in closed economies) as a relevant theorem describing the mechanism of the agricultural market. A hypothesis has been adopted that the usability of this theorem is still high. The word "still" refers to repealing an assumption regarding the closed economy. Currently, we have the wheat market which is integrated and subjected to globalisation processes. It changes assumptions regarding the conditions of the price cobweb theorem.

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JEL Classification: *C02, C62, C65*

1. Introduction

One of the most relevant issues is the mechanism of price shaping and its impact on choices made by these entities. The point here is particularly the impact of the price mechanisms on the producers' decisions with given choices made by consumers. It is important in the case where producers are price takers. Such conditions are most often characteristic of agricultural producers. Moreover, also an assumption on the relative homogeneity of the product (homogeneous basic products) refers to them. It, in turn, implies similar effect for the market of choices made by consumers and producers, and consequently – almost typical establishment of relations between the demand and supply curves. The specificity of activity of agricultural producers is also the fact that their production is of seasonal and reproducible nature. It is accompanied by a typical postponement of the production effect i.e. the time which lapsed between making a decision and obtaining an effect. This phenomenon takes place for reasons independent from the producer himself, but is of strictly biological nature and results from the specific nature of the course of the agricultural production process in the given geographical latitude. In this period of postponement, economic conditions may change. In the conditions of the market price determination it means that there can be a difference between market prices of products at the time of making a decision and their products at the time of completing the production process (moment of obtaining the production effect, final supply volume). It is determined by many factors related to the environmental-biological-climatic

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determination of nature of production processes in agriculture. It is also related to the above-mentioned economic conditions and seasonal cycles. Moreover, we should also mention the impact of political and regulatory factors which, to a greater or smaller extent distort the natural market mechanism. Finally, we should also include individual psychological predispositions of producers (behavioral factors[3] – part of economic psychology or behavioral economics[4]), including so-called herd behaviour (group thinking syndrome).

In the economics of agriculture is a long tradition to study and specify principles regarding price variability in relation to the aforementioned conditions or to isolate the impact of individual price change determinants. Seasonal and economic principles of price changes enjoy the greatest popularity. In fact, it is more important to get an answer to a basic question – how do agricultural producers respond to these price changes or to forecasts on potential price changes and their levels? One of such possibilities is the "price cobweb theorem" (model). Originally, it was formulated in the conditions of domination of national economies as relatively closed systems with the additional share of external turnover. Currently, market entities in Poland function in the system of the integrated EU market which is characterised by being opened to the global market. Thus, the objective of the studies was to verify the assumptions of the cobweb model in the current market conditions. A hypothesis has been adopted that the usability of the cobweb theorem is still high and its assumptions are useful in interpreting relations between prices and the supply of offered commodities (supply of wheat in the Polish market).

2. Market conditions and the cobweb theorem

In the economics, a lot of attention is devoted to price fluctuations in markets. Their level is a result of the game of demand and supply (markets' striving for achieving the level of equilibrium). These adaptation processes in markets are illustrated by the cobweb model. The price cobweb theorem relates to an analysis of a mutual relation between the price and supply over time. First and foremost, the time refers to the response of supply. Here, we can identify three situations:

1) In the very short period (several months) – the price is the function of demand;
2) In the short period (e.g. 1 year of the given production and commercial cycle) – there are adaptation processes of supply under the existing involvement of production factors (manufacturing potential, the supply may increase thanks to its better use);
3) In the long period (several years) – the supply may increase in relation to the increased involvement of factors (increased production capacity) as a result of investments.

[3] Classical economics (A. Smith, D. Ricardo, J.S. Mill) referred to psychologism while Neo-Clas-sical economics consciously departs from it for the benefit of the logic of choice. However, human behaviour and decision-making processes should not be analysed only on the basis of the theory of economics, particularly Neo-Classical. People are often guided by emotions or are not careful enough in striving for obtaining full information at the moment of making decisions.

[4] The *homo oeconomicus* model was subject to modifications extending between two research approaches: psychologism and antipsychologism (economic psychology, behavioural economics).

The cobweb theorem refers mainly to the first two conditions. This model has been formulated at the same time by: Hanau (1928), Ricci (1930), Schultz (1930) and Tinbergen (1930). The term "cobweb" was used by Kaldor (1934) for the first time, while in the theory of economics this model was more broadly explained by Ezekiel (1938). Ezekiel described a mechanism adjusting the supply and demand using present or delayed prices in the market. The proposed approach as a way of explaining the reality enjoyed great interest in the economics of agriculture and in the economics (AL-Daami and AL-Hiyali, 2017). In Poland, the cobweb model in agriculture was applied for the first time by Schmidt and Mandecki in 1933.

The cobweb model allows to interpret price effects related to shifting the production effect in time, in fact – the impact of the supply level on the price in relation to the price, at which a production decision was made (Finkenstadt, 1995). It is important in the case of such agricultural markets as the cereal market (homogeneous and standardised) with the production and supply effect which is clearly postponed in time. After the accession of Poland to the European Union, the Polish cereal market as well as other agricultural markets is a part of the integrated European market. In the past, these were the conditions of the domestic market with the given supply volumes and on the other hand – with the given demand volume and also with the specific information effectiveness of the market. In the cobweb model, it was implicitly assumed that these decisions have herd conditions (i.e. all response in the same way to a given price signal).

The supply volume is a flexible variable. It is determined not only by the national production but also by the easily available import. As far as the demand is concerned, the situation is similar – the export as part of the EU market significantly increases the scale and flexibility of this variable. This flexibility is also determined by the fact that there are currently much better possibilities of storing agricultural products. Currently applied technologies are also more advanced and transport of products is so efficient and effective that the flexibility of supply and demand significantly increases. Moreover, we should also take into account the hypothesis of rational expectations and the fact that in fact agricultural producers may correct errors made in the past. They have quicker access to market information and latest market forecasts (also those on prices). The aid of agricultural advice is much more developed. Moreover, widely used are supply forward contracts or the increasingly developing form of production organisation for contracts concluded. The above-mentioned reasons and beneficial changes may reduce the usability of the cobweb model in agricultural markets. We should also add the formal aspect of the original approach to this theorem which applied to linear forms of the demand and supply functions. The linearity of these two variables is currently replaced by non-linear relations (Brock and Hommes, 1997). Currently, in addition to the more static approach, we also have a dynamic approach.

3. Essence of the price cobweb model

It is assumed that the cobweb model illustrates the mechanism of achieving market equilibrium. In a traditional form, it was one of the simplest dynamic economic models describing changes

in prices and supply of products for the given demand in the individual market. The name or the term "cobweb" is derived from the shape of the lines. The objective of the model is to determine such a price path for a given commodity so that for each period the demand is balanced by the supply. Various oscillations describing price changes were identified. We deal with periodic oscillations when the demand flexibility is equal to the supply flexibility. On the other hand, we talk about explosive oscillations when the demand flexibility is lower than the supply flexibility. In the last two cases, the market is unstable. The dynamic cobweb model presents price changes and changes in the demand and supply volume in a period longer than one production and commercial cycle. These changes are characterised by the cyclic movement, regular fluctuations with alternating periods of rise and fall. The cobweb model points to the dynamics of the market mechanism which at the time of inequilibrium strives for eliminating both the demand and supply surplus (Fig. 1) and for returning to the state of relative equilibrium. Then, there are e.g. damped fluctuations of prices and quantity of products in the market which means that the market returns to equilibrium when the demand flexibility is higher than the supply flexibility (Lichtenberg and Ujihara, 1989; Onozaki, Sieg and Yokoo, 2000; Włodarczyk, 2006; Lenart and Mazur, 2017; Mazur, 2018).

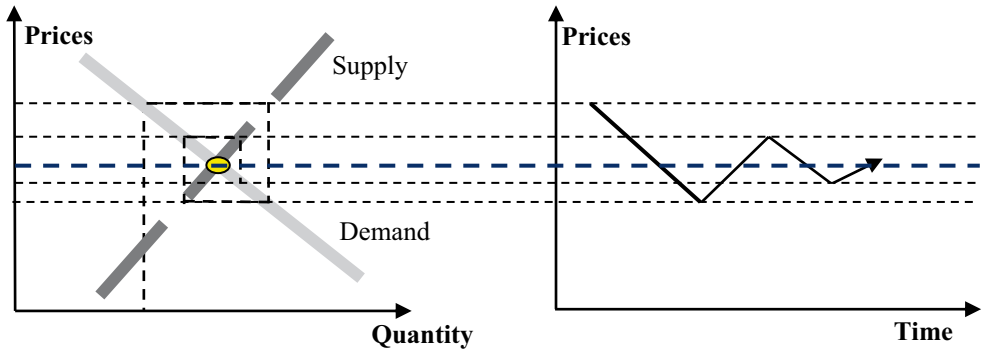


Fig. 1. Dynamic cobweb model (damped oscillations, stable market)

4. Analytical approach to the price cobweb model

Originally, in the cobweb model the functions of demand and supply in relation to the price were linear functions and had the following analytical form:

$$D_t = a - b * P_t, \tag{1}$$

$$S_t = c + d * P_{t-1}, \tag{2}$$

where: D_t – demand volume, S_t – supply volume, P_t – price, a, b, c, d – parameters, t – time.

The above relations illustrate one of the basic assumptions of the cobweb model. It is assumed that the demand is shaped by current prices (in the t period) and the supply – by previous prices (delayed by one period ($t - 1$)). This assumption was adopted based on observations of decisions of customers (demand side) who purchase products, raw materials or services on a basis of current prices in the market. In turn, producers (supply side) make decisions on the

production volume in the previous period and thus based on prices delayed by one period. The prerequisite to achieve equilibrium is presented by the following relation:

$$D_t = S_t \tag{3}$$

After making a conversion including the previous relations, we have:

$$a - b * P_t = c + d * P_{t-1}. \tag{4}$$

It is assumed that the producer making a decision assumes that the price in the period of implementing the production (supply) is the same as in the period of making a decision ($P_t = P_{t-1}$). This price is called the $\overline{P}$ equilibrium price (this can be defined as a projected price) and is used to solve the equation:

$$a - b * \overline{P} = c + d * \overline{P} \Rightarrow \overline{P} = \frac{a - c}{b + d}. \tag{5}$$

In practice, such an equilibrium price may not occur. The type of fluctuations is dependent on the value adopted by the relation:

$$\frac{-d}{b} \quad \text{(where: } b \text{ and } d \text{ are parameters).} \tag{6}$$

The fluctuations can be: (1) damped – if $d < b$; (2) explosive – if: $d > b$; (3) regular – if: $d = b$. Damped fluctuations take place where the market, with every subsequent period, approaches the state of achieving equilibrium. Regular fluctuations occur when the market oscillates around the equilibrium price (where once the price is higher than the equilibrium price by the value and then the price is lower than the equilibrium price by the value). Explosive fluctuations take place where the market, with every subsequent period, goes away from the state of equilibrium.

In the classical cobweb model, producers make decisions on the volume of their future production based on current prices. However, producers may, as it is assumed in the modification of the approach, estimate their production based on the price (hereinafter designated as P_t^e) projected in the future period (Finkenstädt 1995). Then the model assumes a new form:

$$S_t = c + d * P_t^e, \tag{7}$$

where: P_t^e is illustrated by the formula:

$$P_t^e = P_{t-1}^e + w(P_{t-1} - P_{t-1}^e). \tag{8}$$

Here, the parameter is a coefficient of expectations included within the range of $\langle 0; 1\rangle$ (Jakimowicz 2010). Where $w - 1$ then the projected price in the t period is equal to the price in the $t - 1$ period. Looking at the above formula, we can convert it as follows:

$$P_t^e = (1 - w)P_{t-1}^e + wP_{t-1}. \tag{9}$$

In turn, when: $w = 0$ then $P_t^e = P_{t-1}^e$.

Linear functions of the demand and supply do not always reflect the reality properly. For example, in a situation where producers have to incur some fixed costs. Therefore, non-linear functions have been proposed, both for the demand and for the supply:

$$D_t = \frac{a}{P_t}. \tag{10}$$

According to this formula, the demand is a reciprocal of the price, corrected by the a parameter. In turn, the non-linear function of the supply is as follows:

$$S_t = \text{arctg}(\lambda * P_t^e), \tag{11}$$

where: λ is a non-negative parameter regulating the supply function (Hommes 1991).

5. Verification of the model using empirical data

Empirical data came from the IAFE-NRI database prepared based on data of the Statistics Poland. It has been assumed that the demand volume is determined by the total of domestic consumption and export. The supply volume is a total of production, import and initial stocks. Thus, the assumption regarding the closed economy has been repealed in empirical studies. This analysis refers to partial equilibrium (one commodity market) and is also a static analysis (Table 1). Data have been collected in form of time series (2004–2016). The functions of trends for the demand, supply and wheat price have been identified. As it results from Fig. 2, the better adjustment was obtained using the exponential function (demand) and power function (price). Only for the supply, the linear function best explained relations and illustrated the trend over time.

Table 1. Correlation between the wheat supply volume in the "*t*" and its price in the "*t* − 1" period

Specification	Years	Supply in the "*t*" period	Price in the "*t* − 1" period
Years	1.00	0.85	0.73
Supply in the "*t*" period	0.85	1.00	**0.61**
Price in the "*t* − 1" period	0.73	**0.61**	1.00

The formula No. 1 was verified based on the empirical data.. However, it was not confirmed in the case of the data analysed here (Fig. 3). The price rise did not reduce the demand and even stimulated it to some extent. One of potential reasons for this phenomenon could be the growing export of wheat in the analysed period. It should be noted that the positive effect of the accession of Poland to the EU (May 1st, 2004) was a gradual improvement in the trade balance in the wheat market. This process was best visible at the turn of 2014 and 2015, when the domestic consumption was 8,443 thousand tones while the export was 4,921 thousand tones cereals.

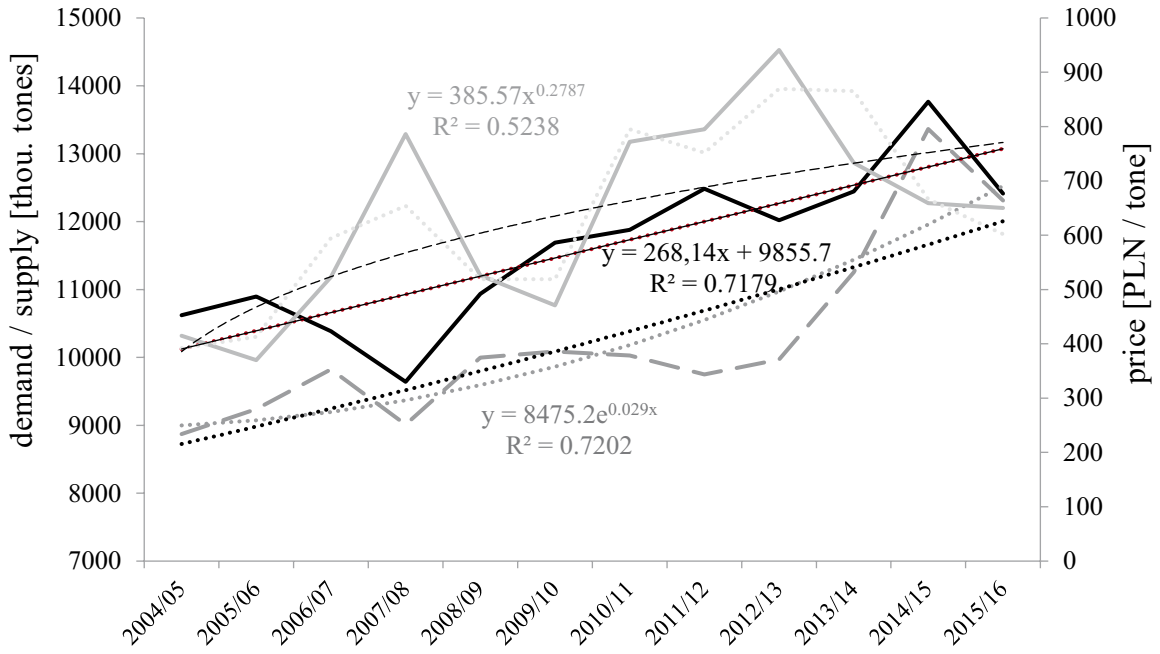
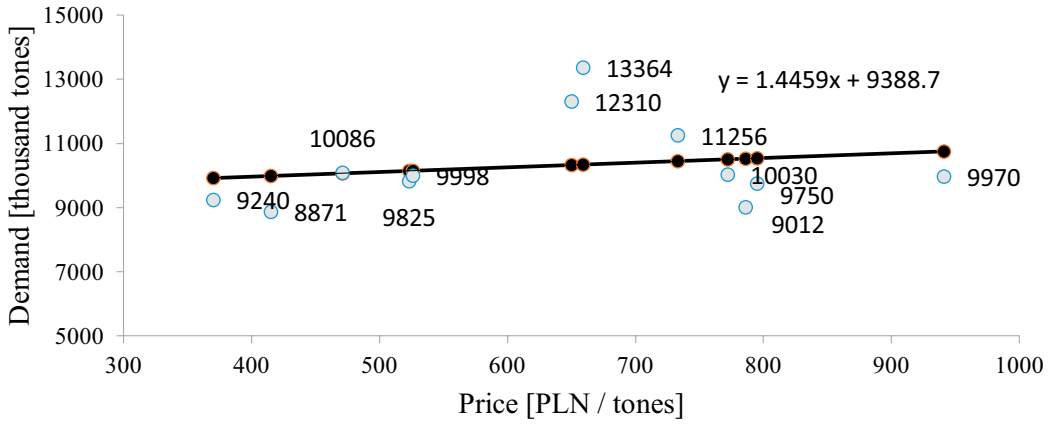


Fig. 2. Functions of trends for the demand, supply and wheat price in Poland (2004–2016)

The supply was analysed using the above-mentioned formula No. 2 (referring to the classical cobweb based on the linear function). The results of estimation are shown in Table 2. The λ and w values in Table 2 optimise the result, i.e. they maximise R^2 and also minimise the standard error. If we are to use methods using the expected price P_t^e, we should determine its initial value, as in the formula No 6 (and 7) referred to P_{t-1}^e. In this paper, the following was adopted:

$$P_{t_0}^e = \frac{S_{t_0-1} * P_{t_0-1}}{S_{t_0}}.. \tag{12}$$



* grey colour marks real data, black colour marks data estimated based on the formula No. 1

Fig. 3. Volume of demand for wheat in relation to the wheat price in the years 2004–2016

It was also assumed that at the beginning of the analysed period there was market equilibrium i.e. $S_{t_0-1} * P_{t_0-1} = S_{t_0} * P_{t_0}$ which after converting leads to the formula No. 12.

Table 2. Results of estimating the wheat supply function in Poland in the years 2006–2016

Specification	Approach_1	Approach_2	Approach_3	Approach_4
Coefficient of determination ($\boldsymbol{R}^2$)	32.79%	99.44%	74.87%	99.79%
Standard error	995.91	924.91	608.99	561.54
Parameter ($\boldsymbol{\lambda}$)	–	0.0064	–	0.0028
Coefficient of expectations ($\boldsymbol{w}$)	–	–	0.1994	0.1602

The approach No. 4, where during statistical verification the best parameters were obtained (satisfactory level of the coefficient of determination and of the standard error), turned out to be the most appropriate one. The coefficient of determination (R^2 showed the high level of explaining the variability of supply by the selected statistical model (Approach_4). The estimated model took the following form: $S_t = 12186.12 * \text{arctg}(0.0028 * P_t^e)$.

Therefore, it can be concluded that in making decisions on the production volume for the next year agricultural producers were most often guided by the level of the price forecast for the next year ($t + 1$) rather than by the current market price (t). This phenomenon was in contradiction with the idea and assumptions of the price cobweb model.

Conclusions

In the economics, much attention is devoted to price fluctuations in markets. Their level is a result of the game of demand and supply and in particular of markets' striving for achieving the level of equilibrium. These adaptation processes in markets are illustrated by the cobweb model where the dynamising factor is the delay of supply in response to the current price. Producers usually respond with a shift in time by one period ($t + 1$). Its objective is to determine a proper price path for this good so that the supply and demand are fully implemented in each year (i.e. market clearing price)[5]. This path can be written as follows: $\{P(t)\}_{t=0}^{\infty}$.

The agricultural sector was an original example showing the mechanism of functioning of the price cobweb, although now, due to a possibility of storing most agricultural products, this principle refers only to products which are perishable or unfit for storage. Therefore, the scope of use and suitability of the model in current economic (market) conditions is reduced.

Agricultural producers, due to the duration of the production process, are somehow forced to make decisions in conditions of uncertainty as to the future price. Therefore, they build on the level of the price from the previous period. Moreover, they usually, do not take into account their previous experiences (the occurrence of short market memory and continuing to make the same errors).

[5] Non-linear deterministic models appeared in economics in the mid-20th century. They are based on an assumption that a source of fluctuations and instabilities, observed in the economic reality, can be not external distortions but endogenous factors. The possibility of using the theory of chaos in economics was mentioned for the first time in 1975. Since that moment, chaos has been found in many existing models of economic phenomena (Orzeszko, 2005; Prokhorov, 2008).

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